# Unified Model of Quasi-Periodic Oscillations

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**Abstract.** We present a new theoretical framework for interpreting observed spectral/temporal characteristics of accreting neutron star and black hole systems as gravity wave (g-mode oscillations). This model successfully incorporates features of earlier models (published by the present authors and colleagues over the last several years) into a more general scheme that reduces in one limit to a classic treatment by Chandrasekhar, placing this paradigm in the tradition of his analysis. It goes beyond his treatment in the inclusion of radial dependence, the incorporation of MHD, and the application to X-ray timing phenomenology. The conceptual picture that goes with this idea is one in which the problem of disk accretion onto a (symmetrical) black hole is the starting point; accretion in geometries where symmetries are broken by magnetic fields is treated with extensions or perturbations to that case. Primary emphasis is on understanding QPO features and power spectrum breaks. Pairs or groups of QPOs that evolve in correlated ways are treated as splittings of eigenfrequencies in a fluid dynamics analysis rather than, say, as beat phenomena. One particular QPO is identified with the Kepler (gravitational) frequency and the other QPOs are related to that one. Because the Kepler frequency is Newtonian and not a General Relativistic effect, the entire treatment is Newtonian, but this helps explain how certain relationships appear to extend over $\sim$ six orders of magnitude in frequency, linking white dwarfs, neutron stars, and black holes in a single comprehensive picture. The explanatory range of the theoretical framework is considerable: it addresses the magnetic field strength and configuration near the compact object, the extension of the Keplerian disk near the central object (and the location of the transition between Keplerian and non Keplerian flow), the presence of advection flow along with disk accretion and the conditions for shock formation in the accretion flow. Successes of earlier treatments, for example fitting the correlated drifts of as many as six persistent power density spectrum features (QPOs or breaks) with minimal parametrization are retained in the new unified scheme. Presented calculations are aimed at (i) extending the explanatory range of the model, (ii) working out details and consequences of the new framework that unifies it with Chandrasekhar's analysis, (iii) making it explicitly an MHD model and not simply hydrodynamics, and (iv) validating it with test. The goal is to have a theoretical synthesis of the existing QPO phenomenology that will serve as a starting point for future X-ray timing observations.

## I. INTRODUCTION

Kilohertz quasi-periodic oscillations (kHz QPOs) have been discovered by the Rossi X-ray Timing Explorer (RXTE) in a number of low mass X-ray binaries (Strohmayer et al. 1996). The existence of two observed peaks with frequencies $\nu_K$ and $\nu_h$ in the upper part of the QPO spectrum became a natural starting point for modeling the phenomenon. Attempts have been made to relate $\nu_K$, $\nu_h$ and the peak difference frequency $\Delta\nu = \nu_h - \nu_K$ with the neutron star spin and possible Keplerian motion of hot matter surrounding the star. Most of the models are based on the interpretation that one of the kHz QPO frequencies is an orbital frequency in the disk [see, for example, the discussion regarding the origin of Keplerian oscillations in Miller, Lamb & Psaltis (1998); TLM98, TO99]. Now, twenty sources have shown kHz QPOs. Sometimes only one peak is detectable, but 18 of these sources have shown two simultaneous kHz peaks. Tables 2 and 3 in van der Klis (2000) summarize the results. As

van der Klis points out there is a remarkable similarity in QPO frequencies and peak separation across a great variety of sources. The recent discoveries of twin QPOs in black hole candidate (BHC) sources GRO J1655-40 and GRS 1915+105 (Strohmayer 2001) further undermine the beat frequency paradigm which relies on the presence of a firm (NS) surface. The correlation between high frequency (lower "kHz" frequency) and low frequency (broad noise component) QPOs previously found by Psaltis, Belloni & van der Klis (1999), hereafter PBK, for BH and NS systems has been recently extended over two orders of magnitude by Mauche (2002) to white dwarfs (WD) binaries (see Fig. 1). With the assumption that the same mechanism produces the QPO in WD, NS and BH binaries, Mauche argues that the data exclude RP, BF models, as well as any model requiring either the presence or absence of a stellar surface or a strong magnetic field.

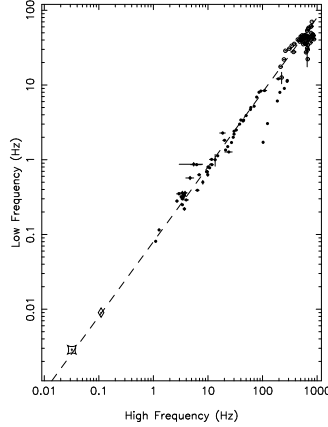Titarchuk & Wood (2002), hereafter TW02, interpret this low-high frequency correlation over 6 order magni-

**FIGURE 1.** Correlation between frequencies of QPO and noise components white dwarfs [diamonds for SS Cyg and squares for VW Hyi(Mauche 2002), neutron star (open circles) and, black hole candidate (filled circles) sources. Neutron star and black hole data are from BPK.

tude and they argue that the observed correlation is a natural consequence of the Keplerian disk flow adjustment to the innermost sub-Keplerian boundary conditions near the central object. This in effect is a common feature for a wide class of systems, starting from WD binaries up to BH binaries. TW02 identify the high frequency as the Keplerian frequency at the adjustment radius (outer radius in the transition layer) and the low frequency as the magnetoacoustic oscillation frequency in the transition layer.

## II. WHY UNIFY QPOS? HOW?

Black holes are central, fundamental, but hard to study.

- BH have no pulsations: QPOs are the best signature.
  - Alternatives to timing : spectra, jets, maybe someday imaging; none obviously better than QPOs.
- Then, there is need to validate understanding of BH QPOs on other sources
  - use neutron stars and white dwarfs; try to bring BH, NS, WD within one comprehensive QPO theory.
- Approach it as a standard fluid problem
  - fluid accreting onto compact object (WD, NS, BH)
  - no new fluid physics peculiar to this problem.
- It is a classical treatment, in rotating frame;
  - use families of QPOs as the data; treat as QPO "spectroscopy", explain the frequencies;
- treat QPOs mainly as eigenvalues of fluid dynamics problems;
  - no beats here; only eigenvalue relationships;

- make model rich enough to treat worst cases (NS cases) (sometimes there are MHD effects).

## III. FLUID DYNAMICS APPROACH:

- QPOS are treated as fluid modes. We perform a stability analysis of these modes.
- Classical antecedents are treatments of similar problems by Rayleigh (1883) and by Chandrasekhar (1961); this analytic treatment is in that tradition. Coriolis, gravitational forces and pressure gradients are important elements in the dynamics.
- There is also a crucial Transition Layer, where azimuthal disk flow changes from Keplerian to sub-Keplerian, because of the central object.
- Generality: treat disks around WD, NS, BH.
- Many QPO modes are known. Model should apply to the richest sets of QPOS, from Sco X-1, other Z and atoll sources.

## IV. QPO PROBLEM−FITTING MOST CHALLENGING CASES

### Observational Facts
QPO modes in Z sources exemplify most of the possibilities. They are the richest set.

- Six QPO or break features; each to be correlated with model features related to fluid modes.
- PDS features have somewhat different names in the updated versions of the model (see e.g. TLM98, TO99, TW02, T03).
- Nomenclature of features as used here:
- Definitions of features Gravitational (Kepler), K; Hybrid, h (horizontal mode); Low, L (vertical mode)=HBO freq, 2L; Viscous, V (MA oscillations); Break, b (involves exponential shots) (see Fig. 2 and §VI).
- This set of QPOs is complete enough to serve for modeling (see Fig. 2).
- It extends to BHs, but not all modes are seen, only some in BHs (so "called HBO" frequencies $\nu_L$ and $2\nu_L$ are not seen in BHs).
- All these modes vary in correlated ways, which a model should explain.
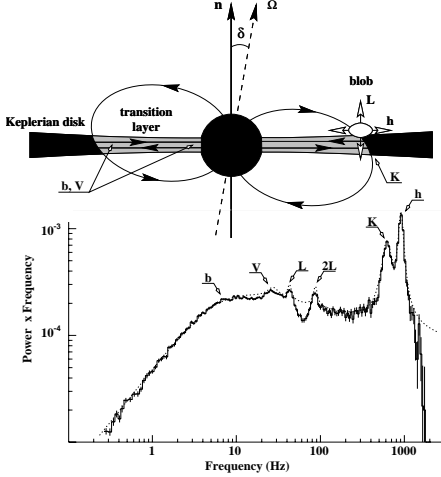- Comparisons (from source to source) are another goal for modeling.

**FIGURE 2.** *Top*: Schematic picture illustrating the idea of the Transition Layer Model. The outflow of the fluid elements forming at the outer boundary of the transition layer oscillates in the equatorial plane (h-mode) and also in the plane which is perpendicular to the disk (L-mode). *Bottom*: Power spectrum of Sco X-1 in units of $F \times P(F)$, $(rms/mean)^2$ labeled with frequencies predictable using the TLM (TOK). The total integration time is $\approx 10$ ks. The best-fit model spectrum is shown with a dashed line.

## V. TRANSITION LAYER CONCEPT

Titarchuk, Lapidus & Muslimov (1998), hereafter TLM98, formulated and solved the problem of the adjustment of the Keplerian disk to the inner boundary condition at the neutron star surface (or at the magnetospheric inner boundary). the radial motion in the disk is controlled by the friction and the angular momentum exchange between adjacent layers, resulting in the loss of the initial angular momentum by accreting matter. The corresponding radial transport of the angular momentum in a disk is described by the equation (see, e.g. SS73)

$$\dot{M}\frac{d}{dR}(\omega R^2) = 2\pi\frac{d}{dR}(W_{r\varphi}R^2), \qquad (1)$$

where $\dot{M}$ is the accretion rate and $W_{r\varphi}$ is the component of a viscous stress tensor that is related to the gradient of the rotational frequency $\omega$, namely,

$$W_{r\varphi} = -2\eta HR\frac{d\omega}{dR}, \qquad (2)$$

where $H$ is a half-thickness of a disk and $\eta$ is the viscosity. The nondimensional parameter that is essential for equations (1-2) is the Reynolds number for accretion flow, $\gamma = \dot{M}/4\pi\eta H$, which is the inverse of the $\alpha-$parameter in the SS73 model. Equation $\omega = \omega_0$ at $R = R_0$ (NS radius, or the innermost radius for a BH), $\omega = \omega_K$ at $R = R_{out}$ were assumed by TLM98 to be

boundary conditions. The solution of equations (1-2) satisfying the above boundary conditions is equation (10) in TLM98 and the equation (see TLM98, Eq. 11)

$$3\theta_{out}/2 = D_1\gamma r_{out}^{-\gamma} + 2(1 - D_1)r_{out}^{-2} \qquad (3)$$

determines $r_{out} = R_{out}/R_0$ as a function of $\gamma-$parameter, where $\theta_{out} = \omega_k/\omega_0$ and $D_1 = (\theta_{out} - r_{out}^{-2})/(r_{out}^{-\gamma} - r_{out}^{-2})$. The adjustment of the Keplerian disk to the sub-Keplerian inner boundary creates conditions favorable for the formation of a hot plasma outflow at the outer boundary of the transition layer (TLM98), because the Keplerian motion (if it is followed by sub-Keplerian motion) must pass through the super-Keplerian centrifugal barrier region. Thus, according to the transition layer model (TLM), the Keplerian oscillator has two branches characterized by high frequency $\nu_h$ ($\sim 1$ kHz) and by low frequency $\nu_L$ ($\sim 50 - 100$ Hz).

In the case when the effects of the pressure gradient can be neglected (Titarchuk 2003, hereafter T03) the frequency $\nu_L$ depends strongly on the angle $\delta$ between the normal to the disk and $\Omega$, the angular velocity of the rotational configuration surrounding the central object (see Figure 1). T03 shows that the hecto-hertz frequencies (detected from 4U 1728-34, 4U 0614+09) can also be identified as the low branch frequency when the pressure effects are important. They depend on the ratio of higher and low "kHz" frequencies (see T03 for details). In the lower frequency part of the QPO spectrum ($\sim 10$ Hz), the second oscillator of the TLM describes the physics of the viscous transition layer, namely, radial viscous oscillations with frequency $\nu_V$ (previously called the "extra noise component") and the diffusive process in the transition region (the innermost part of the disk) which is characterized by the break frequency $\nu_b$.

According to the TLM, all frequencies (namely $\nu_h$, $\nu_L$, $\nu_b$ and $\nu_V$) have specific dependences on $\nu_K$ at the adjustment radius $R_{out}$. *The variation of these frequencies with the disk mass accretion rate (or the disk luminosity) is predictable from the fluid physics of the disk.*

## VI. SUMMARY TABLE OF MODES

$$\text{Mode K} \quad \nu_K = \frac{1}{2\pi}\left(\frac{GM}{R^3}\right)^{1/2}, \qquad (4)$$

$$\text{Mode h} \quad \nu_h = [\nu_K^2 + 4(\Omega/2\pi)^2]^{1/2}, \qquad (5)$$

$$\text{Mode L} \quad \nu_L = (\Omega/\pi)(\nu_K/\nu_h)\sin\delta, \qquad (6)$$

In fact, we can rewrite equation (6) with the angle $\delta$ as a function of only the observed frequencies $\nu_K$, $\nu_h$ and $\nu_L$ (see Fig. 3):

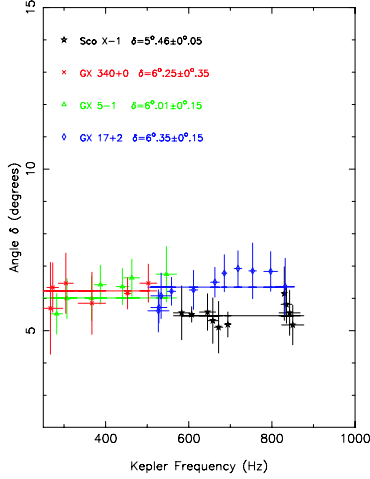$$\delta = \arcsin\left[(\nu_h^2 - \nu_K^2)^{-1/2}(\nu_L\nu_h/\nu_K)\right]. \qquad (7)$$

**FIGURE 3.** Inferred angles $\delta$ between rotational configuration above the disk (possibly NS magnetosphere) and the normal to the plane of Keplerian oscillations as a function of the Kepler frequency for Z-sources: Sco X-1 (black), GX 340+0 (red), GX 5-1 (green), GX 17+2 (blue). Constant $\delta-$ value is consistent with the data. The $\delta-$angle (eq.[7]) is calculated using the lower and higher kHz peaks $\nu_1$, $\nu_2$ and HBO frequencies $\nu_{HBO}$ (see text).
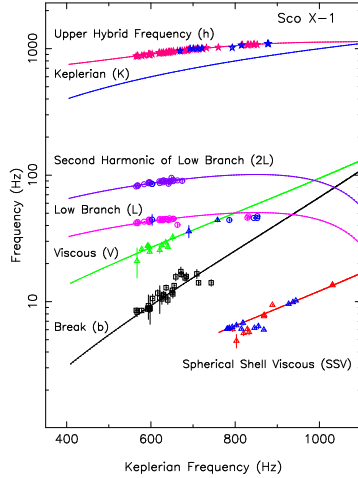


**FIGURE 4.** Classification of QPO branches (hybrid $\nu_h$, Keplerian $\nu_K$, low branch $\nu_L$ and $2\nu_L$, viscous $\nu_V$, break $\nu_b$ and spherical shell viscous $\nu_{ssv}$) in the Z source Sco X-1. This classification is an extended version of the QPO classification for Sco X-1 in TOK. The plot includes QPOs from TBGF's RXTE observations (blue star, circle, and triangle symbols) and 6 Hz observations that represent viscous frequencies under spherical accretion.

$$\text{Viscous (MA) Mode V} \qquad \nu_V \propto \gamma/(r_{out} - r_0), \qquad (8)$$

$$\text{Break Mode b} \qquad \nu_b \propto \gamma/(r_{out} - r_0)^2. \qquad (9)$$

## VII. BLACK HOLE AND NS QPOS, FROM VIEWPOINT OF FUTURE OF X-RAY TIMING

- This picture views QPOs primarily as fluid dynamics under extreme conditions — *an important enough task for a successor mission to RXTE.*
- The analytic treatment is a good beginning, to find QPO frequencies (RMS intensities can be addressed) (see Fig. 4).
- Black holes are the starting point and also simplest, most elegant case, while neutron stars have symmetry- breaking.

### VIII. CONCLUSIONS

- Delta invariant fit in the most challenging Z-source cases – no QPO "left behind"(see Fig. 3).
- Bridging from WD to NS to BH (see Fig. 1). There is continuity of the physics treatment QPOs from BH to WD.
- Other effects (not all covered in this brief summary): - damped harmonic oscillator analog - geometrical picture (covering by disk) related to detectability for high frequency QPOs and NS spins (outside burst), - relation between QPO frequencies and spectral index covered in references below.

## ACKNOWLEDGMENTS

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